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To cite this article: Melissa C. Gilbert (2016) Relating aspects of motivation to facets of mathematical competence varying in cognitive demand, The Journal of Educational Research, 109:6, 647-657, DOI: 10.1080/00220671.2015.1020912

To link to this article: http://dx.doi.org/10.1080/00220671.2015.1020912

Published online: 29 Jun 2016.

Article views: 87

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Relating aspects of motivation to facets of mathematical competence varying in cognitive demand

Melissa C. Gilbert

Stanford Center for Opportunity Policy in Education, Stanford University, Stanford, California, USA

ABSTRACT

The author investigated the relationship between aspects of student motivation and performance on mathematical tasks varying in cognitive demand relevant to meeting the expectations of the Common Core State Standards for Mathematics (CCSS-M). A sample of 479 primarily Latino middle school students completed established survey measures of motivation and a constructed response assessment of two facets of mathematical competence. The assessment measured students' progress toward performing a procedure and demonstrating understanding by providing a written critique of a peer's work, a more cognitively demanding facet. As predicted, higher interest and efficacy in mathematics, lower performance-avoidance goals, and fewer experiences of negative emotions related to performance levels for both facets, while utility and mastery-approach goals (i.e., focusing on understanding mathematics) related only to the more cognitively demanding facet. Implications of these findings for preparing students to be successful mathematical learners, especially in the many states implementing the CCSS-M, are discussed.

Mathematics educators in the United States have long argued that knowing mathematics entails demonstrating multiple skills and understandings as well as engaging in academic behaviors such as justification, problem solving, and communication. Indeed, Curriculum and Evaluation Standards for School Mathematics (1989) and Principles and Standards for School Mathematics (2000) from the National Council of Teachers of Mathematics (NCTM) and Adding It Up (2001) from the National Research Council describe in detail this comprehensive view of mathematical competence. Recently, this view has received significant endorsement with the adoption of the Common Core State Standards for Mathematics ([CCSS-M]; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010) in 45 of 50 states as well as the District of Columbia, four territories, and Department of Defense schools.

The CCSS-M include both Standards for Mathematical Content (SMC) and Standards for Mathematical Practice. The SMC are somewhat analogous to more traditional state content standards organized by grade level for K–8 and by course for high school. However, in a comparison study, Porter and colleagues found that the SMC place a greater focus than prior state standards on demonstrating understanding, an aspect of cognitive demand that expects students to engage in mathematical behaviors such as communicating new mathematical ideas and explaining relationships between concepts (Porter, McMaken, Hwang, & Yang, 2011). For example, a Grade 2 standard in the content domain of Number and Operations in Base 10 states that students “Explain why addition and subtraction strategies work, using place value and the properties of operations” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This compares with a California Grade 2 state standard in the Number Sense domain: “Students estimate, calculate, and solve problems involving addition and subtraction of two- and three-digit numbers” (California State Board of Education, 1999).

A comprehensive view of mathematical competence is also represented in the Common Core Standards for Mathematical Practice (SMP). The SMP, summarized in Table 1, are explicitly based on the five NCTM (2000) process standards (e.g., problem solving, reasoning and proof, communication) and the five strands of mathematical proficiency (e.g., strategic competence, conceptual understanding, adaptive reasoning) described in Adding It Up (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). They describe “varieties of expertise that mathematics educators at all levels should seek to develop in their students” across grades and courses (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 6). In the SMP, students are asked to demonstrate their understanding in ways that go beyond the expectations in the SMC. For example, SMP3 asks that students not only justify their thinking and communicate with others about mathematics but also respond to the mathematical work of others. SMP6 further adds an expectation of students’ precision in communication as well as in calculations. This enhanced rigor is a key shift from prior state standards highlighted in CCSS-M materials (e.g., http://achievethecore.org/category/419/the-shifts). To meet this...
Table 1. Common Core State Standards for Mathematical Practice.

<table>
<thead>
<tr>
<th>Eight Standards for Mathematical Practice (SMP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>Students &quot;start by explaining to themselves the meaning of a problem and looking for entry points to its solution… They monitor and evaluate their progress and change course if necessary… If they continually ask themselves ‘Does this make sense?’&quot; (p. 6)</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>Students &quot;make sense of quantities and their relationships in problem situations…&quot; (p. 6)</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>Students &quot;justify their conclusions, communicate them to others, and respond to the arguments of others.&quot; (p. 6)</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>Students “can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace…” (p. 7)</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>Students &quot;consider the available tools when solving a mathematical problem… They are able to use technological tools to explore and deepen their understanding of concepts.&quot; (p. 7)</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>Students &quot;try to communicate precisely to others… They calculate accurately and efficiently…” (p. 7)</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>Students &quot;look closely to discern a pattern or structure…” (p. 8)</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
</tr>
<tr>
<td>Students “notice if calculations are repeated, and look both for general methods and for shortcuts… They continually evaluate the reasonableness of their intermediate results.” (p. 8)</td>
</tr>
</tbody>
</table>

Source: Common Core State Standards for Mathematics (2010).

The next section provides the rationale for and description of the study. The subsequent section discusses prior research regarding the relationships between the aspects of motivation and facets of mathematical competence used in the study. The method for the present study is then described.

**Motivation and mathematical competence**

Prior research shows that the facets of mathematical competence vary in their cognitive demand and challenge for students (e.g., Porter et al., 2011; Stein, Smith, Henningsen, & Silver, 2000). Stein et al. and Porter et al. discussed levels of cognitive demand in terms of the mathematical thinking required. For example, memorize was the lowest of the five levels used by Porter and colleagues, followed by perform procedures, demonstrate understanding, conjecture, and solve nonroutine problems. Research indicates that students experience greater difficulty solving more cognitively demanding tasks. For instance, they struggle more with demonstrating their understanding of a concept than with performing procedures involving that concept (Evans & Houssart, 2004) though both are important in prior state standards and the CCSS-M. Porter et al. found that the SMC place a lesser emphasis on performing procedures (44% of the standards) than did prior state content standards (49%). More importantly, they place a greater emphasis on demonstrating understanding (36% and 29% of the SMC and prior state standards, respectively). The SMC are thus more cognitively demanding than prior state standards.

High cognitive demand is also reflected in the SMP. As mentioned previously, one expectation, reflected in SMP3 and SMP6, is that students provide a precise written critique of a peer’s work. When students meet this expectation, they are, at minimum, demonstrating their understanding and performing at the third of the five levels of cognitive demand used by Porter et al. (2011). Other SMP expect students to perform at even higher levels by applying mathematics to solve problems that they encounter in everyday life (SMP4) and recognizing patterns (SMP7).

The higher the cognitive demand, the greater the likelihood that students will need the expertise described in SMP1, that is, to make sense of problems and persevere in solving them. Multiple aspects of motivation are associated positively or negatively with the behaviors entailed in SMP1. Students’ sense of efficacy in mathematics is associated with persistence, task engagement, and academic success (e.g., Marsh, Koller, Trautwein, Lüdtke, & Baumert, 2005; Middleton & Spanias, 1999; Pajares & Graham, 1999; Wigfield & Cambria, 2010). Students who endorse mastery-approach achievement goals, that is, focus on learning and understanding the content, report more frequent use of deeper metacognitive and self-regulation strategies, effort, and persistence (e.g., Anderman & Wolters, 2006; Maehr & Zusho, 2009).

Similarly, students who report greater interest in mathematics are more engaged and persistent (e.g., Hidi & Harackiewicz, 2000) and report more cognitive strategy use and self-regulation (e.g., Middleton & Spanias, 1999). Utility, or perceived usefulness of mathematics, is important because of its relationship to choosing to engage in and persist with mathematics (e.g., Fennema, 1989; Hart & Walker, 1993; Eccles & Wigfield, 2000).
Students’ negative emotions in mathematics class need to be minimized in order for students to demonstrate mathematical proficiency since such emotions are inversely associated with students’ ability to focus on and solve mathematical tasks (e.g., Op ‘t Eynde, De Corte, & Verschaffel, 2006). Performance-approach and -avoidance goals are also problematic since their focus on social comparison is negatively associated with persistence and deep strategy use (Maehr & Zusho, 2009). Thus, students who see mathematics as more interesting and useful more believe they are capable of doing mathematics, and focus more on learning and understanding the subject than on their performance relative to others, and report fewer experiences of negative emotions related to the subject would be expected to engage more in the behaviors needed to demonstrate SMP1.

What is less clear from prior empirical research is whether aspects of motivation differentially relate to facets of mathematical competence that vary in their cognitive demand. Few studies in the prior motivation-related research literature differentiate among facets of mathematical competence. A notable exception is the work of Stipek et al. (1998) that separately correlated aspects of upper elementary school students’ self-reported motivation with their learning on “procedurally oriented” versus “conceptually oriented” items on a constructed-response fractions assessment (p. 475). Stipek et al. found that multiple aspects of motivation related to learning at a lower level of cognitive demand, performing a procedure, but none related to learning at a higher level of cognitive demand, demonstrating understanding by modeling mathematical ideas using representations. These findings provide useful insights regarding aspects of motivation that relate to facets of mathematical competence at a lower level of cognitive demand but leave unanswered whether certain aspects relate to facets at a higher level of cognitive demand since Stipek et al. found no significant motivational correlates.

**Current study**

The current study and hypotheses built on prior research in two main ways. First, measures of motivation included in this study were taken from established, psychometrically sound, and reliable scales used in prior research (Eccles & Wigfield, 1995; Linnenbrink, 2005; Midgley et al., 2000). The most relevant prior study (Stipek et al., 1998) analyzed motivational relationships across multiple facets of mathematical competence but used motivation measures that were problematic both conceptually and psychometrically. Conceptually, items were selected that resulted in the confounding of items from different constructs such as positive emotions and interest. Items were categorized into constructs a priori but the appropriateness of the categorization was not confirmed. In addition, the reliability of the scales used in their study was low (e.g., .67 for interest, .59 for efficacy, .65 for mastery orientation, .41 for performance orientation) compared to the reliability for the comparable scales used in the current study (i.e., .95 for interest, .84 for efficacy, .85 for mastery-approach goals, .83 for performance-approach goals, .76 for performance-avoidance goals).

Second, this study provided an important extension of prior work that has considered motivational relationships to facets of mathematical competence. One way the study extended the existing research was by assessing student progress toward each facet of mathematical competence and developing and using a coding procedure for student responses that captured their progress. Thus, the subsequent analyses were not limited to comparisons of students who provided completely correct versus incorrect responses, the approach taken by Stipek et al. (1998). Instead, this study took a more nuanced approach and distinguished between incorrect responses that showed progress toward each facet of mathematical competence and those that did not.

This study also extended prior work by examining two facets of mathematical competence that are needed to meet the expectations of the CCSS-M. Similar to Stipek et al. (1998), students’ ability to perform a procedure, a less cognitively demanding facet of mathematical competence, was examined. Different from Stipek et al., the more cognitively demanding facet considered in this study was students’ ability to demonstrate their understanding by providing a precise written critique of a peer’s work. Though precisely communicating *one’s own* reasoning and listening to a peer’s ideas have been encouraged in prior national documents (e.g., National Council of Teachers of Mathematics, 2000), the SMP within the CCSS-M, particularly SMP3 and SMP6, formally expect students to develop expertise that integrates and extends these practices to critiquing others’ work.

Given students’ difficulty with explaining their *own* thinking (Evens & Houssart, 2004), the cognitive demand and challenge would appear to be higher for students to first understand a peer’s work, then critique it, and finally communicate that critique precisely in writing (presumably, in a way that the peer will also understand the critique). It is likely that active, sustained engagement in mathematical thinking and cognitive strategy use will be needed to demonstrate this challenging facet of mathematical competence. Thus, this study assessed progress toward two facets of mathematical competence expected of students in the CCSS-M that varied in cognitive demand, namely, performing a procedure and demonstrating understanding by providing a critique of a peer’s work.

Hypothesis 1: Certain aspects of motivation associated with the behaviors entailed in SMP1 will relate to performance levels for both facets of mathematical competence. Specifically, students who perform at higher levels for performing a procedure and providing a precise written critique of a peer’s work would report higher interest and efficacy and lower performance-avoidance goals and negative emotions than would those who perform at lower levels.

Hypothesis 2: Certain aspects of motivation associated with the behaviors entailed in SMP1 will relate to performance levels only for the more cognitively demanding facet, providing a precise written critique of a peer’s work. Specifically, students who perform at higher levels for this facet of mathematical competence would report higher mastery-approach goals and utility and lower performance-approach goals than would those who perform at lower levels.

The subsections below briefly review each of the aspects of motivation that informed the hypotheses. The subsections
consider task values (i.e., interest and utility), efficacy (an ability belief), achievement goal orientations, and negative emotions.

**Task values**

Eccles and her colleagues’ expectancy-value model (e.g., Eccles et al., 1983; Wigfield & Cambria, 2010) explains student engagement in mathematical tasks in terms of task values such as interest (or intrinsic value) and an ability belief they refer to as expectancies (see next section). Individuals’ perceived value of a task and their beliefs about their capabilities are strong determinants of why they would want to become or stay engaged in a task (Anderman & Wolters, 2006).

**Interest**

In the expectancy-value model, interest is the enjoyment an individual gets from a task. This is similar to notions of interest defined as an individual’s attraction to or liking of a specific domain (e.g., Pintrich & Schunk, 2002). Intrinsic value is assumed to reflect personal interest (e.g., Hidi, Renninger, & Krapp, 2004), defined as a stable liking of the domain developed over time in conjunction with relevant content knowledge (Anderman & Wolters, 2006). Greater personal interest is associated with higher cognitive engagement and persistence (e.g., Hidi & Harackiewicz, 2000; Iben, 1991) and greater cognitive strategy use, self-regulation, and efficacy (Middleton & Spanias, 1999; Pintrich & De Groot, 1990). These behaviors are considered necessary to provide a precise written critique of a peer’s work. Thus, interest is expected to differentiate among performance levels for both facets of mathematical competence examined in this study.

**Utility**

Utility typically refers to how relevant mathematics is perceived to be for an individual’s future, in terms of job plans and everyday life (e.g., Wigfield & Eccles, 2000). Utility value was not a focus of prior research in which facets of mathematical competence were analyzed separately. However, higher perceived utility is directly associated with intentions to take more mathematics courses but not with higher standardized test performance or course grades (e.g., Anderman & Wolters, 2006; Wigfield & Cambria, 2010). Assuming these customary measures of mathematical competence focus primarily on lower levels of cognitive demand consistent with prior state standards, it is expected that utility will not differentiate among performance levels for performing a procedure. At the same time, students with higher perceived utility value for mathematics may want to be able to critique and explain reasoning in order to be prepared for higher-level mathematics courses or to use mathematics in their work or everyday lives. Thus, utility is expected to differentiate among performance levels for providing a precise written critique of a peer’s work.

**Efficacy**

Expectancies are considered efficacy or competence beliefs in the psychological literature (e.g., Bandura, 1997; Midgley et al., 2000) and self-confidence in learning mathematics (e.g., Hart & Walker, 1993) in the mathematics education literature. There is empirical support that efficacy is associated with persistence, task engagement, and overall academic success in mathematics (e.g., Gilbert et al., 2014; Marsh et al., 2005; Middleton & Spanias, 1999; Wigfield & Cambria, 2010). Efficacy is expected to differentiate among performance levels for both facets of mathematical competence examined in this study.

**Achievement goal orientations**

Achievement goal theory (e.g., Ames & Archer, 1988) conceptualizes student motivation for schoolwork by referring to the purposes or reasons why an individual pursues an academic task and what achievement means to the individual (Anderman & Wolters, 2006; Pintrich, 2000a). These goals encompass students’ attributions (e.g., effort, ability) for success and failure based on attribution theory (Weiner, 1985), a common paradigm in mathematics education. Personal achievement goals also consider the standards for comparison that individuals use to gauge their progress and performance. Knowing this standard provides more information about why students engage in mathematics tasks and what achievement means than can be gleaned from only knowing an individual’s attribution patterns.

Two general types of personal achievement goals have been studied, mastery and performance goals. These goals have been further distinguished in terms of approach or avoidance behavior (e.g., Elliot & Harackiewicz, 1996; Pintrich, 2000b). Three personal achievement goals have been empirically studied thus far in mathematics classrooms: mastery-approach, an active focus on learning and understanding the material; performance-approach, a focus on demonstrating competence relative to others; and performance-avoidance, a focus on avoiding looking incompetent relative to others (e.g., Murayama & Elliot, 2009).

Mastery-oriented students are theorized to have an adaptive pattern for responding to success and failure, attributing success to their understandings gained from hard work and attributing failure to not yet understanding or needing to work harder. Mastery-approach goals have consistently been associated with beneficial outcomes such as higher interest and efficacy, positive emotions, attributions of success to effort, selection of more challenging tasks, and more frequent use of deeper metacognitive and self-regulation strategies and persistence (e.g., Anderman & Wolters, 2006; Kaplan & Maehr, 2002; Turner & Patrick, 2004). Mastery-approach goals are expected to differentiate among students’ performance for more challenging facets of mathematical competence such as providing a precise written critique of a peer’s work.

Performance-approach goals have been associated with anxiety, use of superficial learning strategies such as retention, avoidance of help-seeking and challenge, and low levels of persistence (Anderman & Wolters, 2006; Turner & Patrick, 2004). These goals are not expected to differentiate among performance levels for performing a procedure given the absence of a consistent relationship of performance-approach goals to mathematical competence in prior research. Performance-approach goals are expected to differentiate among performance levels for demonstrating understanding by providing a
precise written critique of a peer’s work with students who perform at the lowest levels reporting the highest endorsement of these goals. These goals have been shown to relate to lower levels of persistence and effort so they are negatively related to the behaviors associated with demonstrating more cognitively demanding facets of mathematical competence. For performance-avoidance goals, the consensus is that they are maladaptive (Anderman & Wolters, 2006) as they have been related to behaviors such as avoiding help seeking, cheating, acting out, and self-handicapping in mathematics classrooms (e.g., Patrick, Turner, Meyer, & Midgley, 2003). Although no prior research has examined these specific goals by facets of mathematical competence, it is likely that they negatively associate with all facets based on prior findings that these goals are maladaptive and detrimental to achievement (e.g., Turner & Patrick, 2004). Thus, performance-avoidance goals are expected to distinguish among performance levels for both facets of mathematical competence examined in this study with students who perform at the lowest levels reporting the highest endorsement of these goals.

**Negative emotions**

Students’ emotional reactions to mathematics class experiences typically refer to their feelings while they are physically in mathematics class (e.g., Linnenbrink, 2005, McLeod, 1992). Emotions such as anxiety, irritation, and exhaustion interfere with students’ ability to focus on and solve mathematical tasks (e.g., Op ‘t Eynde et al., 2006) and to perform well (e.g., Ahmed, van der Werf, Kuyper, & Minnaert, 2013; Cates & Rhymer, 2003; Ma, 1999). The expectations for negative emotions are similar to those for performance-avoidance goals. They are predicted to distinguish among performance levels for both facets of mathematical competence.

To summarize, this section has provided the rationale for the two hypotheses for this study by describing how aspects of motivation have related to the facets of mathematical competence examined in this study or to behaviors or practices presumed to be associated with these facets.

**Method**

**Sample and participant selection**

This research was part of a larger study in conjunction with a partnership between the Motivation Assessment Program (MSP-MAP) at the University of Michigan and Teachers Assisting Students to Excel in Learning Mathematics (TASEL-M) based in Orange County, California public secondary schools. While the TASEL-M project surveyed over 14,000 students, this study focused on prealgebra students at two middle schools who completed an additional mathematics assessment designed for this study. The school district obtained consent from families to participate in their ongoing intervention and evaluation efforts and viewed motivational assessments as part of these evaluation efforts.

The participants for this study were 479 students enrolled in prealgebra at one of two middle schools in Southern California whose principals and teachers were willing to participate in this additional data collection. The majority (454, or 95%) of the 479 students in the sample were seventh graders (20 sixth-grade students, 3 eighth-grade students, 2 no grade level reported), evenly divided between boys (242) and girls (237), including 193 (40%) honors course students (46% girls). Ethnicity was available for 96% of the sample. Similar to the school-level ethnic distribution, most students (69%) were Latino or Hispanic, 13% were Caucasian (non-Hispanic), 11% were Asian/Pacific Islander, and 2% were African American. Regarding language proficiency, of the 441 students for whom English learner (EL) status was available, 163 (34%) were classified as EL. Both schools served lower socioeconomic status populations (i.e., an average of 70% of students received free- or reduced-price lunches).

**Procedure and measures**

All participants completed a self-report motivation measure and a mathematics assessment that measured their progress toward performing a procedure and demonstrating understanding by providing a precise written critique of a peer’s work. Trained research assistants first administered the motivation measure and then the mathematics assessment in October 2005 during students’ regular prealgebra classes. Classroom teachers were present in the room while both measures were completed though they remained seated and unobtrusive during administration and did not view the responses.

**Measure of student motivation**

Students were told that the purpose of the study was to find out their thoughts and feelings about the subject of mathematics and their own mathematics class. They were informed that participation was voluntary and that they could discontinue the survey at any time. After explaining the purpose of the survey and the meaning of the response scale, members of the research team read aloud all directions and survey items to all classes. An exploratory factor analysis ensured that the motivation constructs are seven distinct factors. The extraction method was principal-axis factoring with oblimin rotation and minimum eigenvalue 0.99. All items loaded on predicted factors with acceptable cross-loadings (0.35 or below). These factor and reliability analyses supported the use of these sets of scales in subsequent analyses. See Table 2 for scale reliabilities, sources, and sample items. Preliminary analyses also indicated that these scales conformed adequately to the assumptions for multivariate analyses. Distributions for most scales demonstrated minimal departures from normality as the skewness coefficients for all scales except utility (.03) and mastery approach achievement goals (0.74) were below (0.50).

Table 3 summarizes the intercorrelations for the aspects of motivation measured in this study. The pattern of correlations is consistent with other research with these constructs (e.g., Conley, 2012; Linnenbrink, 2005; Midgley et al., 1998), though the relationship between utility and mastery-approach goals is somewhat stronger (r = .64) yet comparable (r = .59) to other studies that have drawn from this population (Conley, 2012).

In summary, these scales assess distinct aspects of motivation and performed adequately from a statistical perspective based on reliability analyses, factor analyses, and examination of correlations.
Mathematics assessment

A constructed-response measure was developed for this study in consultation with one of the TASEL-M prealgebra teachers. This assessment measured two facets of mathematical competence that varied in their cognitive demand. School district partners were able to allot 15 additional minutes of instructional time to accommodate this assessment. The consulting teacher confirmed that ten minutes would be sufficient for the students to complete this one-page assessment after trained research assistants read aloud the directions. No student requested additional time though they were invited to do so as needed.

Content

This assessment focused on the rational number topic of addition of fractions with unlike denominators, a topic expected to be mastered by the end of fifth grade according to the California Content Standards (California State Board of Education, 1999) and the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This topic was selected because prealgebra students should enter the course already mathematically proficient with this content and should therefore be capable of successfully completing an assessment on this topic early in the course within the allotted time.

Table 2. Reliabilities, sources, and sample items for aspects of motivation.

<table>
<thead>
<tr>
<th>Aspect of motivation</th>
<th>Reliability</th>
<th>n</th>
<th>Source</th>
<th>Sample item</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>.85</td>
<td>5</td>
<td>Eccles &amp; Wigfield (1995)</td>
<td>Math will be useful for me later in life.</td>
</tr>
<tr>
<td>Efficacy</td>
<td>.84</td>
<td>6</td>
<td>Midgley et al. (2000)</td>
<td>Even if a new topic in math is hard, how confident are you that you can learn it? (Anchors: not at all confident, somewhat confident, very confident.)</td>
</tr>
<tr>
<td>Mastery-approach goals</td>
<td>.85</td>
<td>5</td>
<td>Midgley et al. (2000)</td>
<td>My main goal in math is to learn as much as I can.</td>
</tr>
<tr>
<td>Performance-approach goals</td>
<td>.83</td>
<td>4</td>
<td>Midgley et al. (2000)</td>
<td>My goal in math is to look smarter than other students.</td>
</tr>
<tr>
<td>Performance-avoidance goals</td>
<td>.76</td>
<td>4</td>
<td>Midgley et al. (2000)</td>
<td>My goal in math is to avoid looking like I can’t do my work.</td>
</tr>
<tr>
<td>Negative emotions</td>
<td>.64</td>
<td>2</td>
<td>Linnenbrink (2005)</td>
<td>How often do you feel (exhausted/irritated) in your math class? (Anchors: never, sometimes, always)</td>
</tr>
</tbody>
</table>

Note: Except as noted in table, response scales were 5-point Likert-type scales labeled with anchors 1 (not at all true for me), 3 (somewhat true for me), 5 (very true for me).

Validity

Validity studies compared scores on this assessment with scores on two widely used multiple-choice measures of mathematical knowledge. The first measure was the California Standards Test (California Department of Education, 2005) administered the previous May. The second measure was a prealgebra readiness test developed by the California State University/University of California Mathematics Diagnostic Testing Project (MDTP; http://mdtp.ucsd.edu/available-tests/middle-school.html) administered a few weeks prior to this mathematics assessment. These studies, described in detail in Gilbert (2014), indicated that this assessment was a valid measure of students’ rational number understanding despite its relatively short length.

For example, one of the validity studies compared scores on a 12-item rational numbers subtest of the MDTP with competence as measured on the mathematics assessment in this study. This subtest primarily measured fluency with addition of unlike fractions and identifying rational number equivalences. Of the students who showed mastery on this subtest (according to the MDTP scoring procedures), 86% correctly performed a procedure to solve an addition of unlike fractions problem on this study’s mathematics assessment. This result indicates that students who earned high...
scores with comparable rational number items also demonstrated their competence on this assessment and provides evidence for this measure’s content validity.

Scoring student performance
I developed a coding scheme designed to capture the breadth and depth of students’ responses to the prompts. One set of codes documented student progress toward performing a procedure to add unlike fractions correctly. Another set of codes documented student progress toward precisely critiquing Mark’s erroneous approach. A preliminary evaluation of a subset of 150 students’ work ensured that the coding scheme was comprehensive. Two trained raters coded separately all performance assessment responses and had a 94.7% level of agreement. All disagreements were successfully resolved during reconciliation meetings.

This analysis of the student responses informed the development of three performance levels for each facet of mathematical competence; extensive, basic, and none. Students who showed that they could completely and correctly perform a procedure to solve an addition of unlike fractions problem exhibited the level of extensive progress toward performing a procedure (PPP) with this content. Students at the basic level of PPP showed progress toward an accurate solution approach (e.g., adding numerators only) but did not successfully solve an addition of unlike fractions problem. Students who attempted a response but showed no accurate steps (typically, adding numerators and denominators across; e.g., \(3/5 + 1/10 = 4/15\)) described the none level of PPP.

Students’ analysis of Mark’s answer was evidence of progress toward providing a precise written critique of a peer’s work (PPWC). Though there were codes available for other legitimate ways to critique Mark’s work (e.g., use of a number line or explaining the unreasonableness of a sum smaller than an addend), students focused on the correct procedural steps in their analysis. An accurate and thorough critique (i.e., extensive level of PPWC) of Mark’s solution to the addition of unlike fractions problem thus included at least two out of three key elements—a common denominator, an equivalent fraction, and adding numerators only. For example, one student at this level explained,

Mark, your answer is wrong because you CANNOT add two different denominators (the bottom numbers) together. First, you would multiply the one and the two by two so that the denominators (bottom numbers) are equal. When adding, only add the top numbers, because the numbers on the bottom do not change.

The first and last sentences of the student’s explanation reference only adding numerators (one key element). The second sentence explicitly references finding a common denominator (second key element). This student also alludes to an equivalent fraction, explaining how to rewrite 1/2 as 2/4 by multiplying by 2/2 (third key element).

Students who accurately mentioned one element (typically, finding a common denominator) were coded as showing a basic level of PPWC about this topic. Students who attempted a response but included no accurate elements (typically, endorsing Mark’s incorrect method) described the none level of PPWC.

Analytic approach
The research question underlying this study was whether aspects of motivation associated with the behaviors entailed in SMP1 differentially relate to facets of mathematical competence that vary in their cognitive demand and challenge for students. It was hypothesized that some aspects of motivation would relate to performance levels for both facets of mathematical competence and others would do so for progress toward a precise written critique (PPWC) but not progress toward performing a procedure (PPP). A multivariate analysis of variance was used to examine the hypothesized relationships.

Results
As can be seen from Tables 4 and 5, the results with regard to both facets of mathematical competence were generally consistent with predictions. Table 4 summarizes the omnibus statistical tests. Table 5 contains the scale means and post hoc comparisons for each motivation measure by performance levels for PPP and PPWC.

Progress toward performing a procedure
Overall, aspects of motivation related to performance levels for PPP as expected, based on Wilks’ lambda, \(F(14, 906) = 4.09, p < .001\). As can be seen from Table 4, univariate tests indicated that interest, efficacy, performance-avoidance goals, and negative emotions significantly related to performance levels for PPP but utility and mastery- and performance-approach goals did not. The means presented in Table 5 show that students at the extensive level of PPP reported significantly higher mean interest and efficacy and fewer experiences of negative emotions than their peers at either the basic or none performance levels. These students also reported lower performance-avoidance goals than those at the none level.

Progress toward a precise written critique (PPWC)
Overall, aspects of motivation related to performance levels for PPWC as expected, based on Wilks’ lambda, \(F(14, 874) = 2.98, p < .001\). As summarized in Table 4, univariate tests

Table 4. Analysis of variance for aspects of motivation by facet of mathematical competence.

<table>
<thead>
<tr>
<th>Aspect of motivation</th>
<th>Progress toward Performing a Procedure (PPP)</th>
<th>Progress toward a Precise Written Critique (PPWC)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(F(2, 459)) (p) (\eta^2)</td>
<td>(F(2, 443)) (p) (\eta^2)</td>
</tr>
<tr>
<td>Interest</td>
<td>8.11 (&lt; .001) .03</td>
<td>5.31 (.005) .02</td>
</tr>
<tr>
<td>Utility</td>
<td>0.44 (.648) .00</td>
<td>2.33 (.099) .01</td>
</tr>
<tr>
<td>Efficacy</td>
<td>12.83 (&lt; .001) .05</td>
<td>9.27 (&lt; .001) .04</td>
</tr>
<tr>
<td>Mastery-approach goals</td>
<td>1.88 (.154) .01</td>
<td>3.76 (.024) .02</td>
</tr>
<tr>
<td>Performance-approach goals</td>
<td>0.66 (.518) .00</td>
<td>0.43 (.648) .00</td>
</tr>
<tr>
<td>Performance-avoidance goals</td>
<td>6.76 (.001) .03</td>
<td>4.34 (.014) .02</td>
</tr>
<tr>
<td>Negative emotions</td>
<td>12.61 (&lt; .001) .05</td>
<td>10.88 (&lt; .001) .05</td>
</tr>
</tbody>
</table>

Note. Results of multivariate analysis for PPP based on Wilks’ lambda \(F(14, 906) = 4.09, p < .001\). Results of multivariate analysis for PPWC based on Wilks’ lambda \(F(14, 874) = 2.98, p < .001\).
indicated that interest, efficacy, mastery-approach goals, performance-avoidance goals, and negative emotions related to performance levels for this more cognitively demanding facet of mathematical competence. Utility also related to performance levels for this more cognitively demanding facet of mathematical competence. Contrary to expectations, performance-avoidance goals, and negative emotions related to performance levels for this more cognitively demanding facet of mathematical competence. The empirical data from this study also support the presumption that students who showed greater progress toward demonstrating understanding by providing a precise written critique of a peer’s work would report a greater endorsement of aspects of motivation associated with SMP1 such as applying cognitive strategies to make sense of their mathematical work, putting forth sustained effort, and persevering through difficulties. Aspects of motivation that relate to these behaviors, namely interest, utility, efficacy, and mastery-approach goals, related to progress for this facet of mathematical competence. As had been hypothesized, mastery-approach goals did not also relate to progress toward performing a procedure, indicating that these goals may be particularly relevant for more cognitively demanding facets of mathematical competence. The inconsistent relationship of mastery-approach goals on some measures of mathematics achievement (e.g., course grades) in prior research (e.g., Linnenbrink-Garcia, Tyson, & Patall, 2008; Maehr & Zusho, 2009) may thus be attributable to variations in the cognitive demand of the facets of mathematical competence that were assessed. This is an important example of how a nuanced consideration of motivation and mathematical competence contributes to the existing research in the fields of motivation and mathematics education.

The lack of significant findings regarding performance-approach goals contributes to an ongoing debate about when these goals are beneficial. Despite association with negative outcomes such as avoidance of help-seeking, debate continues regarding whether there are situations in which these goals are beneficial (e.g., Keys et al., 2012; Linnenbrink-Garcia et al., 2008). The results in the current study suggest that they related to neither facet of mathematical competence. Though the expected negative relationship with progress toward providing

Discussion

This study represents an important extension of prior work relating motivation and mathematics achievement. In addition to using improved measures of motivation as noted earlier, the study provided an assessment of progress toward the facets of mathematical competence considered, and focused on two facets needed to meet the expectations of the CCSS-M (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010).

The findings illuminated differential relationships with aspects of motivation associated with the behaviors entailed in making sense of problems and persevering in solving them (SMP1). The significance of the omnibus tests for both facets indicates that these aspects, as a group of motivation constructs, are important for students to develop the mathematical expertise they will need to be “college and career ready” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010, p. 57) according to the CCSS-M. In addition, the univariate results provide useful insights regarding how specific aspects of motivation relate to progress toward facets of mathematical competence at varying levels of cognitive demand.

The findings regarding performing a procedure suggest that higher interest and efficacy in mathematics, lower performance-avoidance goals, and fewer experiences of negative emotions related to progress toward less cognitively demanding facets of mathematical competence. The findings for interest, efficacy, and negative emotions are consistent with those of Stipek et al. (1998) for this facet. The result regarding performance-avoidance goals is consistent with other studies (e.g., Turner & Patrick, 2004) that have shown negative relationships of this aspect with academic success.

Table 5. Mean motivation differences by levels for each facet of mathematical competence.

<table>
<thead>
<tr>
<th>Measure of motivation</th>
<th>Extensive</th>
<th>Basic</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Progress toward Performing a Procedure (PPP)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>3.30</td>
<td>1.24</td>
<td>2.68</td>
</tr>
<tr>
<td>Utility</td>
<td>4.10</td>
<td>0.83</td>
<td>4.08</td>
</tr>
<tr>
<td>Efficacy</td>
<td>3.63</td>
<td>0.76</td>
<td>3.16</td>
</tr>
<tr>
<td>Mastery-approach goals</td>
<td>4.00</td>
<td>0.90</td>
<td>3.75</td>
</tr>
<tr>
<td>Performance-approach goals</td>
<td>2.64</td>
<td>1.11</td>
<td>2.84</td>
</tr>
<tr>
<td>Performance-avoidance goals</td>
<td>2.16</td>
<td>1.03</td>
<td>2.51</td>
</tr>
<tr>
<td>Negative emotions</td>
<td>2.55</td>
<td>0.92</td>
<td>2.95</td>
</tr>
<tr>
<td>Progress toward A Precise Written Critique (PPWC)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>3.43</td>
<td>1.24</td>
<td>3.08</td>
</tr>
<tr>
<td>Utility</td>
<td>4.27</td>
<td>0.75</td>
<td>4.07</td>
</tr>
<tr>
<td>Efficacy</td>
<td>3.74</td>
<td>0.70</td>
<td>3.47</td>
</tr>
<tr>
<td>Mastery-approach goals</td>
<td>4.19</td>
<td>0.69</td>
<td>3.86</td>
</tr>
<tr>
<td>Performance-approach goals</td>
<td>2.56</td>
<td>1.11</td>
<td>2.73</td>
</tr>
<tr>
<td>Performance-avoidance goals</td>
<td>2.11</td>
<td>0.99</td>
<td>2.27</td>
</tr>
<tr>
<td>Negative emotions</td>
<td>2.34</td>
<td>0.86</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Note. An alpha level of .05 used in all analyses. Sample sizes for Progress toward Performing a Procedure (PPP); Extensive (n = 169), basic (n = 50), none (n = 243). Sample sizes for Progress toward Precise Written Critique (PPWC); Extensive (n = 63), basic (n = 108), none (n = 275). Means in the same row with subscripts a or b differ at p < .05 (two-tailed) in the Tukey honestly significant difference comparison. Means in the same row with subscript z differ at p < .05, one-tailed.
a precise written critique of a peer’s work did not emerge, there is also no evidence that the focus on outperforming peers that has been a key component of performance-approach goals is beneficial for demonstrating mathematical competence.

Recent reconceptualizations of achievement goal theory to distinguish among self-, task- and other-based goals (Elliot, Murayama, & Pekrun, 2011) may also help to explain the surprising findings for performance-approach goals by clarifying the intention of students’ endorsement of performance-approach goals. Continuing to take a nuanced approach to aspects of motivation and to mathematical competence is particularly important to understand whether fostering inter-student competition develops facets of mathematical competence that vary in cognitive demand. The results from this study provide an important first step to building this understanding and suggest that the answer is that they do not.

Implications

This study has important implications for assessing and preparing elementary and middle school students to be successful mathematics learners, especially in the vast majority of states across the United States that will be implementing the CCSSM. These standards expect students to demonstrate multiple facets of mathematical competence, including the two facets examined in this study. Findings from the study provide teachers with important information about how certain aspects of motivation are associated with the behaviors entailed in the first of the SMP. Three important implications for teachers are described here.

First, the findings suggest that students who demonstrate facets of mathematical competence that vary in cognitive demand also endorse particular aspects of motivation that are associated with the sense-making and perseverance highlighted in the first SMP. These aspects have been referred to previously in the literature as adaptive motivation (e.g., Gilbert & Musu-Gillette, 2011). That is, these students not only see mathematics as useful (utility) and see themselves as capable of learning the subject (efficacy), but also find it interesting (interest), focus on learning and understanding the subject (mastery-approach goals) more than avoiding looking dumb (performance-avoidance goals) or outperforming others (performance-approach goals), and report infrequent experiences of negative emotions, such as exhaustion, in mathematics class.

The findings regarding aspects of motivation that related to the more cognitively demanding facet studied, providing a precise written critique of a peer’s work, but not for performing a procedure, are noteworthy for teachers. The results suggest that fostering mastery-approach goals and perceptions of utility in mathematics may be especially important because of the greater challenge posed for students when they have to demonstrate their understanding (Evens & Houssart, 2004). Teachers may want to consider enhancing these aspects of adaptive motivation before expecting students to demonstrate proficiency with this facet of mathematical competence.

A second implication of this study concerns the relationship of aspects of motivation to students’ progress toward providing a precise written critique of a peer’s work. For each of the aspects that related differently by performance level, higher endorsement of adaptive motivation was associated with greater progress to demonstrating this facet of mathematical competence. These results suggest that teacher practices that promote adaptive motivation may be beneficial for developing more cognitively demanding facets of mathematical competence. For example, teachers may find it worthwhile to foster a learning environment in which students are encouraged to listen to and constructively critique each other’s thinking. In this way, teachers can show students both that mistakes are opportunities for learning and that it is important to provide explanations that are precise and complete in order for peers to follow one’s thinking.

A third implication relates to the importance of using assessments that measure students’ progress toward facets of mathematical competence that vary in cognitive demand. In this study, many more students were able to demonstrate extensive progress toward the less cognitively demanding facet than the more demanding facet. Given the varying but overall higher levels of cognitive demand expected in the CCSS-M content and practice standards, teachers need to ensure that their assessments reflect this enhanced rigor. Otherwise, students may be proficient with only some of the dimensions of mathematical expertise needed to meet the standards and be ready for college and careers.

Limitations and future research

There are three important limitations to this study. One limitation concerns the relatively short time (15 min) available for the mathematics assessment. The consulting teacher and I expected that prealgebra students would be familiar with this Grade 5 mathematical content so this time period would be sufficient to provide their responses. The assessment administration protocol provided space for reports of students requesting additional time. The special education students who made this request were not included in this study. However, it is possible that concerns about the time available diminished some students’ performance. This possibility does not affect the main conclusion that aspects of motivation relate to student progress toward facets of mathematical competence that vary in cognitive demand. Future researchers should continue to measure students’ progress toward multiple facets of mathematical competence and provide as much time as possible for students to complete their responses.

Two other limitations are associated with the measures of motivation. First, the scale used to assess negative emotions did not include emotions such as feeling depressed or annoyed that would capture a broader range of negative emotions, and its reliability estimate was lower than the other motivations scales used in the study. Second, some constructs were highly intercorrelated (e.g., utility and mastery-approach goals, \( r = .64 \)) though they factored separately; another study with students from the same overall research project found similarly strong correlations (Conley, 2012; \( r = .59 \)). These relationships were accounted for in the analysis by conducting a multivariate analysis that simultaneously considered the interrelated constructs to focus on the distinctions between, rather than the similarities among, the seven aspects of motivation. However, it is possible...
that these relationships may have masked the importance of some constructs. Future researchers should further examine the interrelationships among aspects of motivation and consider a range of negative emotions.

Conclusions

This study makes two particularly important contributions to research and practice. First, this study demonstrates how aspects of student motivation associated with behaviors expected in the first SMP relate differently to facets of mathematical competence in the CCSS-M that vary in cognitive demand. Certain aspects (e.g., interest, negative emotions) related similarly to students’ progress toward performing a procedure and providing a precise written critique of a peer’s work. Other aspects, namely, endorsement of mastery-approach goals and seeing utility in mathematics, related only to the more cognitively demanding facet.

Second, the approach taken in the study to code student responses on the mathematics assessment allows for an examination of how motivation relates to students’ progress toward facets of mathematical competence that vary in cognitive demand. As teachers support their students’ attainment of the CCSS-M, it is helpful for them to see how enhancing their adaptive motivation is associated with progress toward facets that pose a greater challenge such as providing a precise written critique of a peer’s work.

Future researchers should continue to investigate the relationship of motivation to students’ progress toward facets of mathematical competence that vary in cognitive demand using measures of motivation that draw from the rich body of motivational literature. Doing so will provide researchers, teachers, and students alike with valuable information to help students to be successful mathematics learners and to demonstrate the facets of mathematical competence required to meet the CCSS-M.

Acknowledgments

This manuscript is based in part on a doctoral dissertation (Kevin F. Miller, chair; University of Michigan) and a paper presented at the American Educational Research Association Annual Meeting, April–May 2013, San Francisco, California.

Funding

The National Science Foundation supported the research in this paper as part of the Math and Science Partnership—Motivation Assessment Program (Award No. 0335369) and Teachers Assisting Students to Excel in Learning Mathematics (Award No. 0227303).

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